2017 AMC 10A

Problem 1

What is the value of $(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)+1)$?

表达式$(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)+1)$的值是多少？

$\textbf{(A)}\ 70 \qquad \textbf{(B)}\ 97 \qquad \textbf{(C)}\ 127 \qquad \textbf{(D)}\ 159 \qquad \textbf{(E)}\ 729$

Problem 2

Pablo buys popsicles for his friends. The store sells single popsicles for $\$1$ each, 3-popsicle boxes for $\$2$ each, and 5-popsicle boxes for $\$3$. What is the greatest number of popsicles that Pablo can buy with $\$8$?

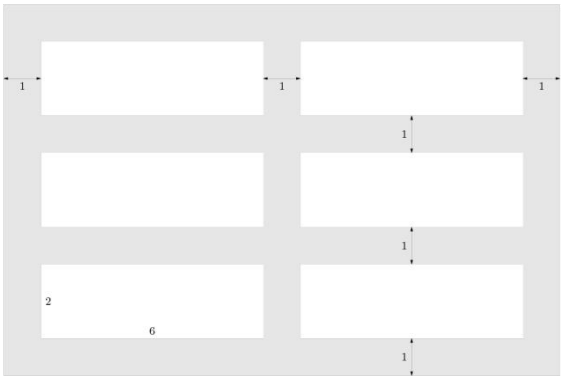
Pablo 给他的朋友买棒冰，商店单根棒冰卖 1 美元一根，1 盒 3 根棒冰每盒卖 2 美元，1 盒 5 根棒冰每盒卖 3 美元。Pablo 有 8 美元，最多可以买多少根棒冰？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Problem 3

Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?

Tamara 的花园里 6 英尺 x 6 英尺规格的花床有 3 行，每行 2 个。如下图所示，花床被 1 英尺宽的走道分隔和包围。走道的总面积是多少平方英尺?



$\textbf{(A)}\ 72 \qquad \textbf{(B)}\ 78 \qquad \textbf{(C)}\ 90 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 150$

Problem 4

Mia is "helping" her mom pick up 30 toys that are strewn on the floor. Mia's mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

Mia 正在"帮助"她妈妈整理散落在地板上的 30 个玩具。Mia 的妈妈每 30 秒钟可以把 3 个玩具放进玩具盒里，但是每次这 30 秒一过，Mia 就从盒子里拿出 2 个玩具。当 Mia 和她妈妈第一次把所有 30 个玩具放入盒子中时，需要多少分钟？

$\textbf{(A)}\ 13.5 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 14.5 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 15.5$

Problem 5

The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

两个非零实数的和是它们乘积的 4 倍。问这 2 个数的倒数之和为多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 12$

Problem 6

Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which of these statements necessarily follows logically?

Carroll 女士承诺任何人只要在即将到来的考试里，做对所有的选择题，此次考试就能得到一个 A。由此，下面哪句话逻辑上是对的？

(A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong

如果 Lewis 没有得到 A，那么他的所有选择题做的都是错的

(B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.

如果 Lewis 没有得到 A，那么他至少有一道选择题做的是错的

(C) If Lewis got at least one of the multiple choice questions wrong,then he did notreceive an A.

如果 Lewis 至少有一个选择题做错了，那么他就得不到 A

（D）If Lewis received an A， then he got all of the multiple choice questions right.

如果 Lewis 得到一个 A，那么他所有的选择题都做对了

（E）If Lewis received an A,then he got at least one of the multiple choice questionsright.

如果 Lewis 得到一个 A，那么他至少做对了一道选择题

Problem 7

Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

Jerry 和 Silvia 想从一块正方形场地的西南角走到东北角。Jerry 先朝东走，再朝北走到达目标，但是 Silvia 朝着东北角直接走直线到达目标。下面那个数更接近，Silvia 所走的路程和 Jerry 所走路程相比少走多少？

$\textbf{(A)}\ 30\%\qquad\textbf{(B)}\ 40\%\qquad\textbf{(C)}\ 50\%\qquad\textbf{(D)}\ 60\%\qquad\textbf{(E)}\ 70\%$

Problem 8

At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

一次聚会有 30 人，其中有 20 人相互之间都认识，还有 10 人任何人都不认识。相互之间认识的人就拥抱，相互之间不认识的人就握手，求共握手多少次？

$\textbf{(A)}\ 240\qquad\textbf{(B)}\ 245\qquad\textbf{(C)}\ 290\qquad\textbf{(D)}\ 480\qquad\textbf{(E)}\ 490$

Problem 9

Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town A to town B, a distance of 10 km all uphill, then from town B to town C, a distance of 15 km all downhill, and then back to town A, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny?

Minnie 在平地上骑车时速度是 20 千米每小时（kph），下山时速度是 30kph，上山时速度是 5kph；Penny 在平地上骑车速度是 30kph，下山速度是 40kph，上山速度是 10kph；Minnie 从 A 镇到 B 镇，总共是 10km 的上山路程，然后从 B 镇到 C 镇，总共是 15 公里的下山路程，之后折返 A 镇，总共 20km 的平地。Penny 反方向走同样的路线。问骑车走完这 45km 的路程，Minnie 比 Penny 多花了多少分钟？

$\textbf{(A)}\ 45\qquad\textbf{(B)}\ 60\qquad\textbf{(C)}\ 65\qquad\textbf{(D)}\ 90\qquad\textbf{(E)}\ 95$

Problem 10

Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

Joy 一共有 30 根薄竹竿，每根竹竿的长度是 1cm 到 30cm 之间的不同的整数长度。她把长度为 3cm，7cm 和 15cm 的竹竿放在桌上，然后她想选择第 4 根竹竿，保证能和这 3 根形成一个面积为正的四边形。剩下的竹竿中，有多少种选择？

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Problem 11

The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

和线段$\overline{AB}$的距离在 3 个单位长以内的所有点，在三维空间中组成的区域的体积为216π。*AB* 的长度为多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Problem 12

Let S be a set of points (x, y) in the coordinate plane such that two of the three quantities $3,\ x+2$, and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description for S?

S 表示这三个量 $3,\ x+2,\ y-4$ 中有 2 个量相等，且第 3 个量不大于这个共同值的坐标平面内所有的点(x, y)所组成的集合。下面哪个是对 *S* 的正确描述？

（A） a single point丨1 个点

（B） two intersecting lines丨2 条相交的直线

（C）three lines whose pairwise intersections are three distinct points丨3 条直线，这 3 条直线两两相交形成 3 个不同的点

（D）a triangle丨1 个三角形

（E） three rays with a common point丨 有一个公共点的 3 条射线

Problem 13

Define a sequence recursively by $F_0 = 0,\ F_1 = 1,$ and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by $3,$ for all $n \geq 2.$ Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$ What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

定义数列：$F_0 = 0,\ F_1 = 1,$且对于$n \geq 2.F_n$等于$F_{n-1} + F_{n-2}$ 除以 3 所得余数．因此数列开头几项为$0, 1, 1, 2, 0, 2, \ldots$，求值 $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 14

Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda?

每周 Roger 会用他的零花钱买一张电影票和一瓶苏打水，上周 Roger 的零花钱是 A 美元。他的电影票的价格是 A 与苏打水价格之差的 20%，而他的苏打水的价格是 A 与电影票价格之差的 5%。Roger 花了 A 的百分之多少买了电影票和苏打水?结果四舍五入到整数

$\textbf{(A)}\ 9\% \qquad \textbf{(B)}\ 19\% \qquad \textbf{(C)}\ 22\% \qquad \textbf{(D)}\ 23\% \qquad \textbf{(E)}\ 25\%$

Problem 15

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloé's number? (Assume they cannot be equal)

Chloe 从区间［0，2017］中均匀随机的选择一个实数，Laurent 独立的从区间［0，4034］中均匀随机的选择一个实数。Laurent 的数比 Chloe 的大的概率是多少？（假设它们不能相等）

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{5}{6} \qquad \textbf{(E)}\ \frac{7}{8}$

Problem 16

There are 10 horses, named Horse 1, Horse 2, . . ., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

一共有 10 匹马，名字分别为马 1，马 2，…，马 10，这是根据这些马沿着圆形赛道跑一圈所需要的分钟数命名的：马 *k* 跑 1 圈需要 *k* 分钟。在初始时间 0 时，所有的马都在跑道的起点，这些马开始朝着同一方向跑，并且各自速度恒定。$S > 0$（以分钟为单位）表示这 10 匹马再次同时处于起点的最短时间，在这里$S = 2520$。$T > 0$（以分钟为单位）表示至少 5 匹马再次同时处于起点的最短时间，那么 *T* 的各位数字之和是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 17

Distinct points P, Q, R, S lie on the circle $x^2 + y^2 = 25$ and have integer coordinates. The distances PQ and RS are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

P，*Q*，*R*，*S* 是圆$x^2 + y^2 = 25$ 上的 4 个不同的点，并且坐标均为整数。*PQ* 和 *RS* 的长度都是无理数。$\frac{PQ}{RS}$的最大可能值是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 3\sqrt{5} \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 5\sqrt{2}$

Problem 18

Amelia has a coin that lands heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $q-p$?

Amelia 有一枚硬币，扔到正面朝上的概率是$\frac{1}{3}$，Blaine 有一枚硬币，扔到正面朝上的概率为 $\frac{2}{5}$。Amelia 和 Blaine 轮流扔各自的硬币直到某个人得到正面朝上。第一个得到正面朝上的人就赢了。每次扔硬币都是独立事件。Amelia 先扔，Amelia 赢的概率是$\frac{p}{q}$，这里 p 和 q 是互质的正整数。那么$q-p$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 19

Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

Alice 拒绝坐在 Bob 或 Carla 的旁边，Derek 拒绝坐到 Eric 的旁边。他们 5 人要坐到一排 5 张椅子上，并且满足这些要求，一共有多少种坐法？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 40$

Problem 20

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

$S(n)$表示正整数 n 的各位上数字之和。例如，$S(1507) = 13$。对于某个特定的正整数 n，$S(n) = 1274$，$S(n+1)$的值可能是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 1239 \qquad \textbf{(E)}\ 1265$

Problem 21

A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

一个边长为 *x* 的正方形内接在一个边长分别 3，4，5 的直角三角形中，满足正方形的一个顶点和三角形的直角顶点重合，一个边长为 *y* 的正方形内接在另一个边长分别是 3，4，5 的直角三角形中，满足正方形的一条边和直角三角形的斜边重合。$\frac{x}{y}$是多少？

$\textbf{(A)}\ \frac{12}{13}\qquad \textbf{(B)}\ \frac{35}{37}\qquad \textbf{(C)}\ 1\qquad \textbf{(D)}\ \frac{37}{35}\qquad \textbf{(E)}\ \frac{13}{12}$

Problem 22

Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC are tangent to a circle at points B and C respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

等边三角形 *ABC* 的边$\overline{AB}$和$\overline{AC}$与一个圆分别相切于点 *B* 和点 *C*。$\triangle ABC$面积有多少在圆的外面？

$\textbf{(A)}\ \frac{4\sqrt{3}\pi}{27}-\frac{1}{3}\qquad \textbf{(B)}\ \frac{\sqrt{3}}{2}-\frac{\pi}{8}\qquad \textbf{(C)}\ \frac{1}{2}\qquad \textbf{(D)}\ \sqrt{3}-\frac{2\sqrt{3}\pi}{9}\qquad \textbf{(E)}\ \frac{4}{3}-\frac{4\sqrt{3}\pi}{27}$

Problem 23

How many triangles with positive area have all their vertices at points (i,j) in the coordinate plane, where i and j are integers between 1 and 5, inclusive?

坐标平面内，有多少个面积为正的三角形，满足所有的顶点在(i,j)，其中 *i*，*j* 都是在 1 到 5 之间，并且包含 1 和 5 的整数？

$\textbf{(A)}\ 2128\qquad \textbf{(B)}\ 2148\qquad \textbf{(C)}\ 2160\qquad \textbf{(D)}\ 2200\qquad \textbf{(E)}\ 2300$

Problem 24

For certain real numbers a, b, and c, the polynomial $g(x) = x^3 + ax^2 + x + 10$ has three distinct roots, and each root of $g(x)$ is also a root of the polynomial $f(x) = x^4 + x^3 + bx^2 + 100x + c$. What is $f(1)$?

a，b，c 为实数，满足：多项式 $g(x) = x^3 + ax^2 + x + 10$ 的三个不同的根都是多项式 $f(x) = x^4 + x^3 + bx^2 + 100x + c$ 的根，求 $f(1)$ 的值。

$\textbf{(A)}\ -9009 \qquad \textbf{(B)}\ -8008 \qquad \textbf{(C)}\ -7007 \qquad \textbf{(D)}\ -6006 \qquad \textbf{(E)}\ -5005$

Problem 25

How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

100 到 999（包含 100 和 999）有多少个整数，有如下性质：各个位上的数字的某些排列形成的数是位于 100 和 999 之间的且是 11 的倍数。例如，121 和 211。

$\textbf{(A)}\ 226 \qquad \textbf{(B)}\ 243 \qquad \textbf{(C)}\ 270 \qquad \textbf{(D)}\ 469 \qquad \textbf{(E)}\ 486$

2017 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	D	B	B	C	B	A	B	C	B	D	E	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	C	B	D	D	C	D	D	E	B	C	A	

2017 AMC 10A Solution

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